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# Brane–worlds in 5D supergravity

Anna Ceresole [⋆] and Gianguido Dall'Agata [♯]

[⋆] Dipartimento di Fisica, Politecnico di Torino and
Istituto Nazionale di Fisica Nucleare, Sezione di Torino
C.so Duca degli Abruzzi, 24, I-10129 Torino.
ceresole@athena.polito.it

[♯] Institut für Physik, Humboldt Universität
Invalidenstraße 110, 10115 Berlin, Germany
dallagat@physik.hu-berlin.de

**Abstract**

We summarise the present status of supersymmetric Randall–Sundrum brane–world scenarios and report on their possible realisation within five–dimensional matter coupled $\mathcal{N} = 2$ gauged supergravity.

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The idea of obtaining phenomenology from more than four dimensional space–times is almost as old as Einstein's theory of relativity. Physically sensible results can arise upon imposing some confining mechanism for particles and interactions in the extra dimensions. Many different such mechanisms have been proposed over the years, and almost all of them have given new insights on the possible structure and features required for any consistent theory in $D > 4$.

The first proposal was the Kaluza–Klein recipe [1], where all the extra dimensions are compactified on some internal manifold, such that vibrations over this space are seen as masses and charges for particles from the four–dimensional point of view. This old idea, first suggested to unify electrodynamics and gravity, is still much used nowadays in strings and supergravities to extract phenomenology from these ten and eleven–dimensional theories by compactifying them on internal manifolds of the size of the Planck length [2].

A new step forward was provided by the Hořava–Witten model [3] (and its five-dimensional realisation [4]). In this model, particles and fields of spin ranging from 0 to 1 are confined to membranes embedded in an eleven–dimensional space with still a compact eleventh direction. This new mechanism opens the possibility to obtain at least one extra dimension of size greater than the usual Planck scale.

Although the constraining on branes is a valid mechanism for spin 0 and 1 fields, one cannot use it for gravity. It is indeed known that gravity couples to all energy sources, and aiming essentially at describing the geometry of the whole space, it cannot be confined to a submanifold only.

Actually, the idea of viewing our universe as some membrane embedded in a higher–dimensional space can be traced back to [5], but for what just said it was left undeveloped at that time.

So, how is it possible to confine spin 3/2 and spin 2 fields on membranes? In spite of the above objections, last year Randall and Sundrum [6] have proposed a mechanism which goes very close to constraining gravity. They show that one can obtain some graviton bound state which is centered and concentrated on a four–dimensional subspace of a five–dimensional bulk. The main new feature of this scenario is that they actually impose the ambient space to have a non–vanishing cosmological constant such that a new mechanism for confining gravity fluctuations can occur. An outstanding result is that the extra dimension involved can be made arbitrarily large and even completely unfolded, without ruining the possibility of having confinement. This implies that one could obtain effective four–dimensional gravity starting from a non–compact five–dimensional space–time.

Since this scenario is claimed to solve many longstanding phenomenological problems, like the cosmological constant value and the hierarchy problem, it seems compelling to find out whether it can be embedded into higher dimensional supersymmetric theories like supergravity and string theory, that provide the most natural candidates for field unification. This would provide for them a more rigorous derivation while avoiding the need of many fine–tunings and the ad-hoc selection of their characteristics.

First of all we would like to distinguish between three different setups which are all called Randall–Sundrum models, but actually have different theoretical relevance.

In the original proposal [6], meant to provide a solution to the hierarchy problem, there

are two membranes located at the orbifold fixed points of a still compact fifth dimension. The complete action includes 5D gravity and sources for the two membranes:

$$S = S_{bulk} + S_{branes}.$$

One serious drawback of the *two–brane scenario* was the presence of a negative tension membrane that however, as was later shown, could be consistently taken to infinity. Thus came the second setup, the *one-brane scenario* [9] with only one singular 'thin' (delta-functional) membrane, where gravity is confined by a volcano potential, surrounded by two slices of AdS space. The metric of such configuration can be put in the form

$$ds^2 = a^2(y)\, dx^2 + dy^2, \tag{1}$$

respecting four–dimensional Poincaré invariance, and where the *warp factor* behaves for $|y| \to \infty$ as $a^2(y) = e^{2A(y)} \sim e^{-k|y|}$ $(k > 0)$.

Due to the infinite extension of the fifth dimension $y$, this model leads to the very intriguing possibility of getting a real substitute for the usual compactification scheme. Therefore this *alternative to compactification* is not only appealing from a phenomenological point of view, but also from a purely theoretical one, since one could try to implement it in supergravity and string theory, replacing the usual Kaluza–Klein mechanism.

Making any of these scenarios compatible with some supersymmetric theory has been attempted in various ways [7, 16] and ultimately requires the implementation of supersymmetry on singular spaces [8]. This state of affairs is still somewhat unsatisfactory, even in the example where you can eliminate the negative tension brane.

The obvious improvement would be to trade the singular source with a smooth ("thick") domain wall solution of some (super–)gravity theory [11], the *thick brane scenario*. In this way our universe could be seen as some natural result of the dynamics of one of the known theories in higher–dimensional space–time. More precisely, due to the presence of a cosmological constant, the theories to be explored for the embedding of such scenarios are the gauged supergravities, and in the simplest instance the five–dimensional ones [13, 14]. Unhappily, all the many attempts made up to now in this direction can be summarised in a no–go theorem [10] stating that no smooth Randall-Sundrum scenario can be found within 5D gauged supergravity interacting with an arbitrary number of vector and/or tensor multiplets. Moreover, even when also hypermultiplets are coupled, one encounters various difficulties that, although not in the form of a no–go theorem, hint to the fact that if trapping of gravity on the brane in 5D supergravity is possible, it certainly must take place in a very specific model. The reason for such rareness stems on the requirement of having a supersymmetric flow joining two stable IR fixed points with the same cosmological constant, lying on each side of the brane. Upon examining a large class of supergravity scalar potentials, these constraints seem to be quite hard to meet.

Possible ways out of this picture are the inclusion of massive multiplets, as suggested by models containing the "breathing mode" [12] or other generalisations of the existing supersymmetric theories, that are presently under investigation.

# 1 $\mathcal{N}=2$, $D=5$ gauged supergravity

It is well known that *gauged* supergravities, where some of the global isometries of the standard theory ($R$–symmetry included) are made local, admit a scalar field potential that, at some critical point, generates the cosmological term. More precisely, the global isometries of the scalar manifold are made local by substituting the ordinary derivatives in the Lagrangean with new covariant derivatives, depending on the vector fields which become gauge bosons of this invariance $\partial_\mu \longrightarrow \mathcal{D}_\mu = \partial_\mu + gA_\mu$. As this process obviously brakes supersymmetry, one has to restore it by adding new terms (shifts), of first order in the coupling constant $g$, in the supersymmetry transformation rules. This will create new contributions in the Lagrangean that are also of first order in $g$, and are interpreted as mass terms for the Fermi fields. To cancel supersymmetry variations of these terms, one has to add to the Lagrangean a further piece of order $g^2$: the scalar potential. The great miracle of gauged supergravities is that, under certain conditions, the process does not go on to infinity as all the higher order terms vanish identically, leaving with a consistent supersymmetric theory[1].

Given that five dimensions are necessary for an effective four–dimensional brane–world, it is natural to look first into the minimal supersymmetric $\mathcal{N}=2$ (*i.e.* with eight real supercharges) supergravity, and in order to avoid the no-go theorem [10], one considers the generic interaction with an arbitrary number $n_V$ of vector, $n_T$ of tensor and $n_H$ of hyper-multiplets [14]. The fermionic fields of this model are two gravitini $\psi^i_\mu$ that are symplectic Majorana spinors ($i=1,2$ are $SU(2)_R$ indices and $\mu=0,\ldots,4$), $n_V+n_T$ gaugini $\lambda^{i\tilde{a}}$ ($\tilde{a}=1,\ldots,n_V+n_T$), and the hyperini $\zeta^A$ with $A=1,\ldots,2n_H$. The bosonic sector is composed, beside the graviton $e^a_\mu$, the graviphoton $A^0_\mu$, the $n_V$ vectors $A^I_\mu$ and $n_T$ tensors $B^M_{\mu\nu}$, by the two sets of scalars $\phi^{\tilde{x}}$ and $q^X$ belonging to the vector and hyper–multiplets, spanning respectively a very special and a quaternionic target space.

Quoting only what is essential to search for smooth RS solutions, the scalar potential is given by

$$\mathcal{V}(\phi,q) = g^2\{2W^{\tilde{a}}W^{\tilde{a}} - \left[2P_{ij}P^{ij} - P_{\tilde{a}ij}P^{ij}_{\tilde{a}}\right] + 2\mathcal{N}_{iA}\mathcal{N}^{iA}\} \tag{2}$$

where the various quantities are the shifts appearing in the supersymmetry rules for the fermion fields, whose relevant parts (in a backgound with vanishing fermions and vectors) are

$$\delta_\varepsilon\psi_{i\mu} = \mathcal{D}_\mu\varepsilon_i + \frac{i}{\sqrt{6}}g\,\gamma_\mu\varepsilon^j P_{ij}, \tag{3}$$

$$\delta_\varepsilon\lambda^{\tilde{a}}_i = -\frac{i}{2}f^{\tilde{a}}_{\tilde{x}}\gamma^\mu\varepsilon_i\,\mathcal{D}_\mu\phi^{\tilde{x}} + g\varepsilon^j P^{\tilde{a}}_{ij} + gW^{\tilde{a}}\varepsilon_i, \tag{4}$$

$$\delta_\varepsilon\zeta^A = -\frac{i}{2}f^A_{iX}\gamma^\mu\varepsilon^i\mathcal{D}_\mu q^X + g\varepsilon^i\mathcal{N}^A_i. \tag{5}$$

The above symbols are defined as

$$P_{ij} \equiv h^I P_{I\,ij}, \qquad P^{\tilde{a}}_{ij} \equiv h^{\tilde{a}I}P_{I\,ij}, \tag{6}$$

[1]In a geometric approach, the change in the derivative reflects into a direct modification of the Bianchi identities for the various fields, and the modification of the supersymmetry rules is a natural consequence of their closure.

$$W^{\tilde{a}} = \tfrac{\sqrt{6}}{4} h^I K_I^{\tilde{x}} f_{\tilde{x}}^{\tilde{a}}, \qquad \mathcal{N}^{iA} = \tfrac{\sqrt{6}}{4} h^I K_I^X f_X^{Ai}. \tag{7}$$

in terms of various geometric scalar dependent quantities such as $h^I(\phi), h^{\tilde{a}I}(\phi)$, the vielbeins of the very special and quaternionic manifolds $f_{\tilde{x}}^{\tilde{a}}(\phi)$, $f_{iX}^{A}(q)$, the corresponding Killing vectors $K_I^{\tilde{x}}$ and $K_I^X$, and the quaternionic prepotential $P_{I\,ij}(q)$ [13, 14]. Notice that the $W^{\tilde{a}}$ shift in the gluino susy rule is specifically due to the tensor multiplet couplings [13].

In order to search for supersymmetric configurations, one solves Killing spinor equations that are obtained by setting to zero supersymmetry variations of the Fermi fields. Moreover, these latter also give the supersymmetry flow equations, that are first order differential equations giving rise to solutions of the full equations of motion.

# 2 RS solutions

Solitonic solutions of the theory at hand, with suitable features to yield an RS scenario, must have a metric of the form (1). Moreover, the scalar potential (2) must have at least two different critical points, where the scalar fields $\Phi = \{\phi, q\}$ reach some fixed value $\Phi \to \Phi^*$, such that as $|y| \to \infty$, the metric approaches that of an AdS space, $ds^2 \to e^{-|y|}dx^2 + dy^2$. Therefore the warp factor must approach zero as we go toward this fixed point.

To summarise the outcome of the analysis carried out when only vector multiplets are coupled to supergravity [10], the supersymmetry flow equations around the fixed points yield the 'wrong' behaviour for the fields as functions of the warp factor,

$$\phi^{\tilde{x}}(a) = \phi^{\tilde{x}*} + \frac{c^{\tilde{x}}}{a^2}. \tag{8}$$

Thus, the warp factor turns out to be *always increasing* as we move towards the fixed point, while it should decrease for an RS solution.

The obvious hope is that things could change if we allow for couplings to hypermultiplets.

Let us then examine the susy equations (3)–(5). For BPS solutions to exist, one has to find some Killing spinors which, in addition to the usual constraint coming from setting to zero the gravitino susy rule (3)

$$i\gamma^y \varepsilon_i \sim g P_{ij} \varepsilon^j, \tag{9}$$

also satisfy

$$\delta_\varepsilon \lambda_i^{\tilde{a}} = \mathcal{A}^{\tilde{a}}{}_i{}^j \varepsilon_j = 0 \tag{10}$$

and

$$f_{iA}^X \delta_\varepsilon \zeta^A = \mathcal{B}^X{}_i{}^j \varepsilon_j = 0 \tag{11}$$

where the operator matrices $\mathcal{A}^{\tilde{a}}$ and $\mathcal{B}^X$ can be read off from (4) and (5) respectively.

The key point is that, once the solution to (9) is inserted into (10) and (11), the $\mathcal{A}$ and $\mathcal{B}$ operators reduce to simple SU(2) matrices acting only on the SU(2) index of the Killing spinor. This implies that the possibility of finding BPS solutions amounts to having a Killing spinor eigenvector of $\mathcal{A}^{\tilde{a}}$ (the same is true for $\mathcal{B}^X$) with zero eigenvalue, i.e. the matrix $\mathcal{A}^{\tilde{a}}$ must be degenerate.

A closer look shows that, after making explicit the $SU(2)$ structure, these matrices actually have the same form:

$$\mathcal{A}^{\tilde a}{}_i{}^j \sim iS^{r\tilde a}(\sigma_r)_i{}^j + Q^{\tilde a}\delta_i^j, \tag{12}$$
$$\mathcal{B}^X{}_i{}^j \sim iS^{rX}(\sigma_r)_i{}^j + Q^X\delta_i^j, \tag{13}$$

where $S^{r\tilde a}, S^{rX}, Q^{\tilde a}, Q^X$ indicate some real combination of the various $y$–dependent quantities. Requiring $\det\mathcal{A}^{\tilde a} = 0$ (the same for $\mathcal{B}$), imposes for each $\tilde a$

$$(Q^{\tilde a})^2 + (S^{\tilde a})^2 = 0, \tag{14}$$

which has no solutions except for

$$Q^{\tilde a} = 0 = S^{\tilde a}. \tag{15}$$

These conditions confirm that no RS scenario can be obtained if tensor multiplets are present, since $Q^{\tilde a} \equiv W^{\tilde a}$ is precisely the contribution of tensor multiplets to the gaugino susy rule and thus to the potential. All the other constraints instead lead to some partial differential equations with respect to the $y$ coordinate involving the scalar fields and the warp factor. Such equations may or may not yield appropriate solutions, but cannot be solved without selecting a specific model and computing the relevant geometric quantities. Therefore, the study of the general supersymmetric flow equations does not suffice to clear up the situation, as was the case for vectors and/or tensor multiplets coupled to supergravity.

More recent studies [15] seem to show that there is at least the possibility of finding infra–red fixed points from the hypermultiplet scalar manifold. Indeed, one can find critical points near which the scalars behave as

$$\Phi^i \sim \Phi^{i*} + c^i a(y)^n, \tag{16}$$

for some power $n > 0$. However, this is not enough to give an RS solutions, since we have seen that one needs *two* such points in the same theory, where the value of the potential is the same, and so far none of these have been found.

# 3 Outlook

We have discussed the simplest and most natural supersymmetric framework where smooth Randall–Sundrum brane worlds could fit, that is the fully coupled $D = 5$, $\mathcal{N} = 2$ gauged supergravity. It is perhaps interesting to notice that although unsuccessful up to now, the search for such cosmological scenarios has already stimulated new results. Indeed, it was among the main motivations for constructing the above theory, that was previously known only in absence of hypermultiplets [13].

Further searches bring to the more thorough exploration of specific hyper–matter coupled models, or even to more general extensions. For instance, beside considering more systematically theories with a higher number of supersymmetries, that would change the susy rules, one should look at new possible couplings of the $\mathcal{N} = 2$ theory to massive multiplets that exist as representations of the $SU(2,2|1)$ supergroup.

A promising possibility seem to be to add the interaction with massive *vector* multiplets, since these should be linked to scalars with massess big enough to allow for the appearence of infra–red critical points [16, 15]. The final word is likely to be hidden in the general discussion of $d = 5$ supersymmetric flows/domain–walls, that is now under investigation [15].

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